ON THE NATURE OF SIMULTANEITY

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The relative nature of the simultaneity of two events that occur at the same time in two separate space points is established in the special theory of relativity through a gedanken experiment sometimes known as Einstein's train paradox. Two space-point simultaneity occurs when two events take place at the same time and an observer at the midpoint views the simultaneous arrival of light rays triggered by the two events. In the various explanations offered of this experiment and similar ones, pre-relativistic and incomplete arguments are presented that tend to obscure the facts. A simpler and more detailed consideration based from the very beginning on the special theory of relativity shows that simultaneity is actually absolute. Additionally, defining simultaneity in terms of proper time, simultaneous events in one inertial system of reference occur at the same proper time in any other inertial system of reference.

Following the presentation of [1]Bergmann, consider two very long and thin trains moving relatively to each other as shown in Figure 1. Let S and S' be the frames of reference in which the stationary and the moving trains appear motionless, respectively. At instant $t = 0$, in the S-frame, the two frames of reference precisely coincide and the trains coincide in the space interval $-L_0/2 \le x \le +L_0/2$. At that very same instant two thunderbolts strike simultaneously at the points $x_{2,1} = \pm L_0/2$ of the S frame. It is assumed that the S'-frame moves to the right of S with a speed v.

$$x'_1 = -\frac{L_0}{2\gamma} \quad t'_1 = \frac{v}{c}\frac{L_0}{2\gamma c} \qquad x'_0 = 0 \quad t'_0 = 0 \qquad x'_2 = \frac{L_0}{2\gamma} \quad t'_2 = -\frac{v}{c}\frac{L_0}{2\gamma c}$$

x'
x

$$x_1 = -\frac{L_0}{2} \quad t_1 = 0 \qquad x_0 = 0 \quad t_0 = 0 \qquad x_2 = \frac{L_0}{2} \quad t_2 = 0$$

Figure 1

Applying the Lorentz transformation and setting $\gamma \equiv \left[1-(v/c)^2\right]^{1/2}$, where c is Einstein's constant, it turns out that, in the S' frame, the thunderbolts strike space point $x_2' = +L_0/(2\gamma)$ at instant $t_2' = -(v/c)[L_0/(2\gamma c)]$ and $x_1' = -L_0/(2\gamma)$ at $t_1' = +(v/c)[L_0/(2\gamma c)]$, respectively. It is evident that the thunderbolts do not occur simultaneously in the S' frame. The clocks at points x_2' and x_1' exhibit different times. This is a necessary result of the Lorentz transformation in order to uphold the constancy of the speed of light in the moving frame. It should be observed that both light rays do take the same time interval, $L_0/(2\gamma c)$, to reach the midpoint, x_0', of the moving train. This means that the ray from the right will get to x_0' at instant $(1-v/c)[L_0/(2\gamma c)]$ while the ray from the left will arrive later at x_0', at instant $(1+v/c)[L_0/(2\gamma c)]$, thus the light rays do not arrive in a simultaneous fashion at x_0'.

It is then instructive to determine the point in the moving train, if any, at which the rays do arrive simultaneously. Let ξ' be the position of such a point and τ' the time of arrival as shown in Figure 2, then

$$\tau' = t_1' + (\xi' - x_1')/c = t_2' + (x_2' - \xi')/c \qquad (1)$$

from which it follows that

$$\xi' = -(v/c)(L_0/2\gamma), \quad \tau' = L_0/(2\gamma c) \qquad (2)$$

and when the inverse Lorentz transformation is applied to ξ' and τ' there results

$$\xi = 0, \quad \tau = L_0/(2c) \qquad (3)$$

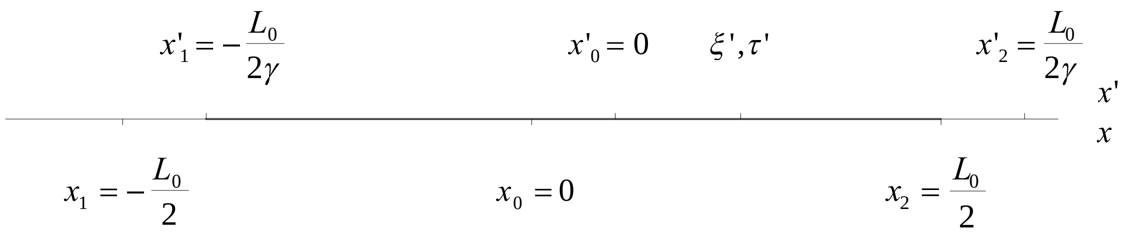

Figure 2

which, of course, happens to be the midpoint , x_0, of the stationary train at the correct arrival time required for the light rays originating from points $x_{2,1} = \pm L_0/2$.

Since the thunderbolts strike at the same time both ends of the stationary train, light signals triggered by the incoming thunderbolts arrive simultaneously at the midpoint of the stationary train. However, since the thunderbolts strike at different times the ends of the moving train it makes no sense to expect the light signals to arrive simultaneously at its midpoint.

Consider now a similar gedanken experiment with the only difference that both thunderbolts strike at time $t = 0$ at the points $y_{2,1} = \pm L_0/2$ of the S frame, as depicted in Figure 3. It is clear that light rays will arrive simultaneously at the origin of the stationary frame defining an event E and, also, at the origin of the moving frame, defining another event E'. However, E and E' will not be simultaneous with respect to each other nor will they be caused by the same pair of light rays. Once again, requiring unconditionally that the light rays arrive simultaneously at the midpoints of their original separations in their respective frames of reference leads to incongruous results. It is easy to see, for example, that event E occurs at $x = 0$, $t = L_0/(2c)$ to which corresponds event E'' in frame S' at $x' = -(v/c)[L_0/(2\gamma)]$, $t' = -(v/c)[L_0/(2\gamma c)]$.

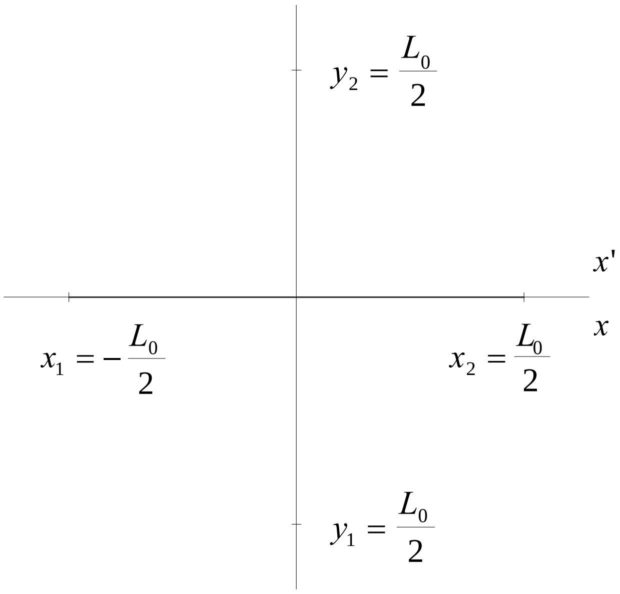

Figure 3

Observers in all frames of reference, at different space points, will record events - which happen to posses two space-point simultaneity in a particular frame- at different times but no immediate conclusion with respect to the order of their occurrence may be reached. For example, passengers of the moving train in positions $x' > \xi'$ will see the light ray from the front end arrive before that from the rear end while for passengers of the moving train in positions $x' < \xi'$ the situation will be reversed. Incorrect interpretations of this fact have needlessly led to postulate "parallel realities" and to question causality.

It is proposed that simultaneity be defined by means of proper time, that is, events are simultaneous when they occur at the same proper time. In the case of Einstein's train paradox it is observed that the thunderbolts strike at the proper time $[(\pm L_0/2)^2-(c*0)^2]^{1/2} = L_0/2$ in the stationary frame and at $\{[\pm L_0/(2\gamma)]^2 - c^2*[\pm vL_0/(2\gamma c^2)]^2\}^{1/2} = L_0/2$ in the moving frame so that the thunderbolts occur simultaneously in both frames. With the proposed definition, simultaneity becomes absolute and the paradox ceases to exist. According to the dictionary definition it could be stated that no moving frame knew of two-space-point simultaneity as that in a stationary one. In other words, the previous two-space-point simultaneity was a unique property of frame S as is, for example, the sphericity of a sphere in frame S.

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[1]Peter Gabriel Bergmann, "Introduction to the Theory of Relativity" (Dover Publications, Inc., New York, 1976), pp. 29-32.